

a/ $V(3,4)$ $W(6,-1)$ $R(-2,4)$ $T(0,-5)$ $U(2,-2)$ ②

b/ $\vec{A} = \overrightarrow{TU} \begin{pmatrix} 2-0=2 \\ -2-(-5)=3 \end{pmatrix}$ $\vec{C} = \overrightarrow{WV} \begin{pmatrix} 6-3=3 \\ -1-4=-5 \end{pmatrix}$

c/ D'après la figure

$$\vec{B}\left(\begin{matrix} 2 \\ 0 \end{matrix}\right) \quad \vec{D}\left(\begin{matrix} 0 \\ 4 \end{matrix}\right) \quad \vec{E}\left(\begin{matrix} -3 \\ 4 \end{matrix}\right) \quad \vec{G}\left(\begin{matrix} -3 \\ -6 \end{matrix}\right) \quad \vec{F}\left(\begin{matrix} 3 \\ 3 \end{matrix}\right)$$

d/ cf figure

e/ $\vec{A} + \vec{B} \begin{pmatrix} 2+2=4 \\ 3+0=3 \end{pmatrix}$ $\vec{C} + \vec{D} \begin{pmatrix} 3+0=3 \\ -5+4=-1 \end{pmatrix}$ $\vec{G} + \vec{F} \begin{pmatrix} -3+3=0 \\ -6+3=-3 \end{pmatrix}$

f/ cf figure

g/ $\vec{W} = \vec{H} - \vec{T} \begin{pmatrix} -4-2=-6 \\ -4-4=-8 \end{pmatrix}$ $\vec{X} = \vec{J} - \vec{K} \begin{pmatrix} 4-(-5)=9 \\ -2-0=-2 \end{pmatrix}$

$$\vec{Y} = \vec{A} + \vec{U} - \vec{D} \begin{pmatrix} 2-1-0=1 \\ 3-1-4=-2 \end{pmatrix}$$

h/ cf figure

i/ $\vec{Q} = 3\vec{F} \begin{pmatrix} 3 \times 3=9 \\ 3 \times 3=9 \end{pmatrix}$ $\vec{R} = 2\vec{B} + 2\vec{A} \begin{cases} 2 \times 2 + 2 \times 2 = 8 \\ 2 \times 0 + 2 \times 3 = 6 \end{cases}$

j/ cf figure

k/ $\vec{S}_1 + \vec{A} + \vec{C} = \vec{0} \Rightarrow \vec{S}_1 = -\vec{A} - \vec{C} \begin{pmatrix} -2-3=-5 \\ -3-(-5)=2 \end{pmatrix}$

l/ cf figure

m/ $\vec{S}_2 + 3\vec{C} - 2\vec{A} = \vec{0} \Rightarrow \vec{S}_2 = 2\vec{A} - 3\vec{C} \begin{pmatrix} 2 \times 2 - 3 \times 3 = -5 \\ 2 \times 3 - 3 \times (-5) = 21 \end{pmatrix}$

n/ $3\vec{S}_3 - 2\vec{D} + \vec{S}_B = \vec{0} \Rightarrow \vec{S}_3 = \frac{\vec{D} - \vec{S}_B}{3} \begin{pmatrix} (2 \times 0 - 5 \times 2)/3 = -10/3 \\ (2 \times 4 - 5 \times 0)/3 = 8/3 \end{pmatrix}$

$$9/ \|\vec{A}\| = \sqrt{2^2 + 3^2} = \sqrt{13} \quad \|\vec{G}\| = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} \quad (3)$$

$$9/ \|\vec{S}\| = \sqrt{(-1)^2 + (-9)^2} = \sqrt{82} \quad \|\vec{C}\| = \sqrt{3^2 + (-5)^2} = \sqrt{34}$$

$$\|\vec{H}\| = \sqrt{(-4)^2 + (-6)^2} = \sqrt{32}$$

$$\|\vec{C}\| + \|\vec{H}\| = \sqrt{34} + \sqrt{32}$$

$$\|\vec{C} + \vec{H}\| = \sqrt{(3 + (-4))^2 + (-5 + (-6))^2} \\ = \sqrt{1 + 81} = \sqrt{82}$$

$$9/ \vec{C} \begin{cases} 2 \times 0,5 = 1 \text{ m} \\ 3 \times 0,5 = 1,5 \text{ m} \end{cases} \quad \vec{F} \begin{cases} -3 \times 0,5 = -1,5 \text{ m} \\ 4 \times 0,5 = 2 \text{ m} \end{cases} \quad \vec{H} \begin{cases} -4 \times 0,5 = -2 \text{ m} \\ -4 \times 0,5 = -2 \text{ m} \end{cases}$$

$$10/ \vec{D} \begin{cases} -1 \times 10 = -10 \text{ N} \\ -1 \times 10 = -10 \text{ N} \end{cases}$$

$$11/ \|\vec{A}\| = \sqrt{(2 \times 0,5)^2 + (3 \times 0,5)^2} = 1,1 \text{ m} = \sqrt{13} \times 0,5$$

$$\|\vec{B}\| = \sqrt{0^2 + 4^2} \times 0,5 = 2 \text{ m}$$

$$12/ \|\vec{F}\| = \sqrt{3^2 + 3^2} \times 10 = 42,4 \text{ N} \quad \|\vec{G}\| = \sqrt{(-3)^2 + (-6)^2} \times 10 = 10 \times \sqrt{45} \\ = 67,1 \text{ N}$$

$$13/ \vec{H} \begin{cases} -4 \\ -4 \end{cases} \quad x_H = \sqrt{32} \times \cos 45^\circ = 4 \\ y_H = \sqrt{32} \times \sin 45^\circ = 4$$

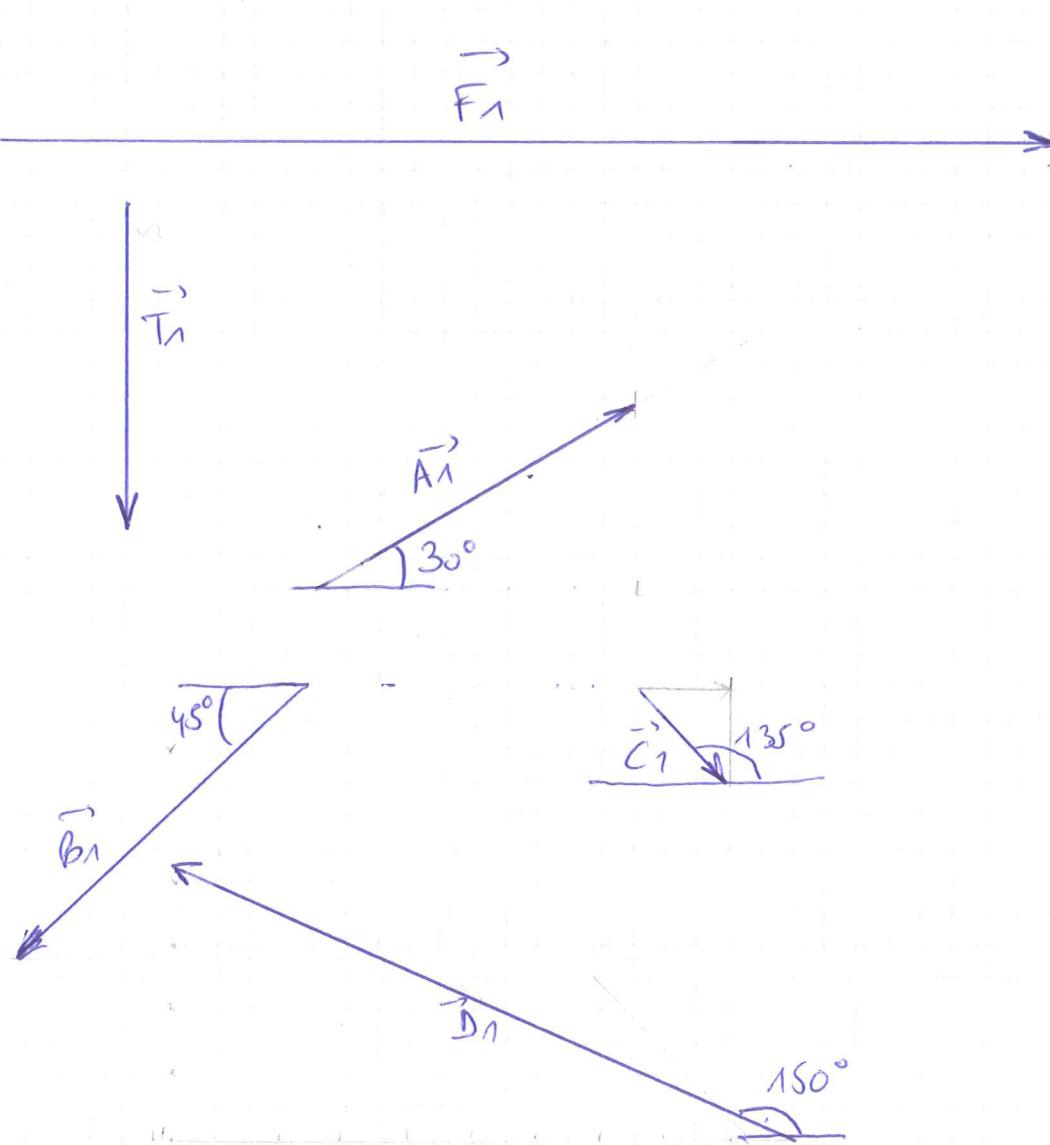
$$14/ \vec{E}_1 = -8 \text{ m} \quad \tan \alpha = \frac{y_E}{x_E} \Rightarrow y_{E_1} = x_{E_1} \cdot \tan \alpha \\ = -8 \times \tan 8^\circ = 1,12 \text{ m} \\ \vec{E}_1 (-8 \text{ m}; 1,12 \text{ m})$$

$$15/ \vec{G}_1 \begin{cases} x_{G_1} \\ y_{G_1} \end{cases} \quad \tan 82^\circ = \frac{y_{G_1}}{x_{G_1}} \Rightarrow x_{G_1} = \frac{y_{G_1}}{\tan 82^\circ} = \frac{-12}{\tan 82^\circ} = -1,7 \text{ m} \\ \vec{G}_1 (-1,7 \text{ m}; -12 \text{ m})$$

$$\star / \quad \begin{aligned} \overrightarrow{C} &= \tan^{-1}\left(-\frac{5}{3}\right) = -60^\circ & \overrightarrow{E} &= \tan^{-1}\left(\frac{4}{-3}\right) = -53^\circ = 127^\circ \quad (4) \\ \overrightarrow{G} &= \tan^{-1}\left(-\frac{6}{-3}\right) = 63^\circ = -117^\circ & \overrightarrow{A} &= \tan^{-1}\left(\frac{3}{2}\right) = 56^\circ \end{aligned}$$

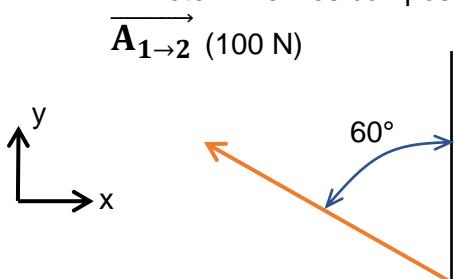
EXERCICE 2. Compléter le tableau ci-dessus et représenter graphiquement les vecteurs forces aux échelles indiquées. Préciser les valeurs de X_F et de Y_F pour chaque cas

Forces	Échelle adoptée	Longueur du vecteur tracé à son échelle	Direction par rapport à l'horizontale \vec{X}	Cordonnées		Module
				X_F	Y_F	
\vec{F}_1	1mm → 5N	170 mm	0°	>0 850	0 0	850 N
\vec{T}_1	1mm → 4N	50 mm	90°	0 0	<0 -200	200 N
\vec{A}_1	1mm → 20 daN	57 mm	30°	>0 987	>0 570	1140 daN
\vec{B}_1	1mm → 5N	60 mm	45°	<0 -212	<0 -212	300 N
\vec{C}_1	1mm → 7daN	15 mm	135°	>0 74	<0 -74	105 daN
\vec{D}_1	1mm → 60 kN	125 mm	150°	-64 95	3750	7500 kN

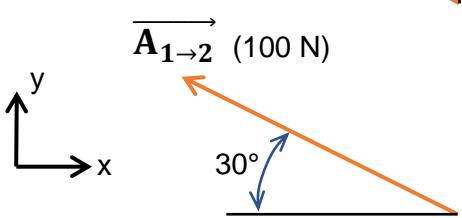


Correction Exo BO - partie 1

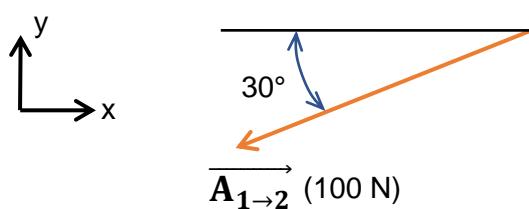
Déterminez les composantes (projections) de la force ci-dessous



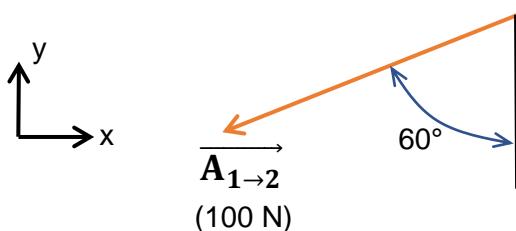
$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 150^\circ \\ 100 \cdot \sin 150^\circ \\ 0 \end{pmatrix}$$



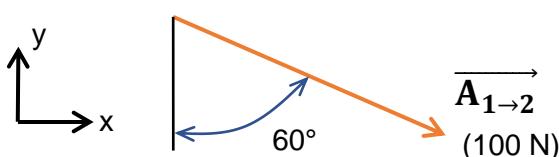
$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 150^\circ \\ 100 \cdot \sin 150^\circ \\ 0 \end{pmatrix}$$



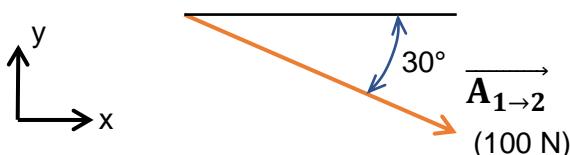
$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 210^\circ \\ 100 \cdot \sin 210^\circ \\ 0 \end{pmatrix}$$



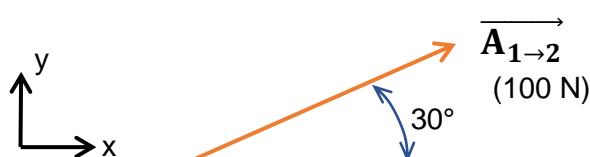
$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 210^\circ \\ 100 \cdot \sin 210^\circ \\ 0 \end{pmatrix}$$



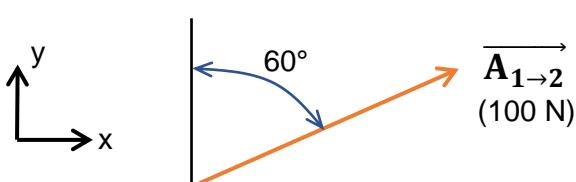
$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 330^\circ \\ 100 \cdot \sin 330^\circ \\ 0 \end{pmatrix}$$



$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 330^\circ \\ 100 \cdot \sin 330^\circ \\ 0 \end{pmatrix}$$



$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 30^\circ \\ 100 \cdot \sin 30^\circ \\ 0 \end{pmatrix}$$

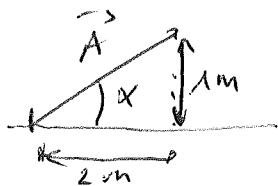
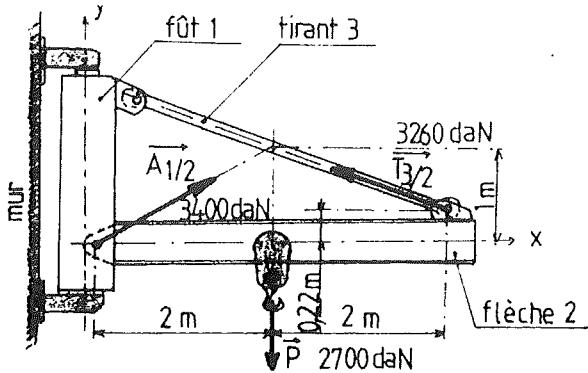


$$\overrightarrow{A_{1 \rightarrow 2}} = \begin{pmatrix} 100 \cdot \cos 30^\circ \\ 100 \cdot \sin 30^\circ \\ 0 \end{pmatrix}$$

(6)

EXERCICE 3.

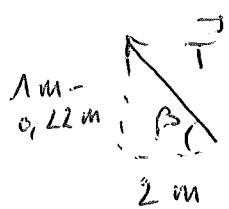
Écrire les coordonnées suivant x et y des forces $\vec{A}_{1/2}$, $\vec{T}_{3/2}$ et \vec{P} . Que peut-on dire de la somme vectorielles $\vec{A}_{1/2} + \vec{T}_{3/2} + \vec{P}$



$$\alpha = \tan^{-1} \frac{1}{2} \Rightarrow \alpha = 26,6^\circ$$

$$X_A = \|\vec{A}\| \cos \alpha = 3600 \cos 26,6^\circ = 3041 \text{ daN}$$

$$Y_A = \|\vec{A}\| \sin \alpha = 3600 \sin 26,6^\circ = 1520,5 \text{ daN}$$



$$\beta = \tan^{-1} \left(\frac{1-0,2^2}{2} \right) \Rightarrow \beta = 21,3^\circ$$

$$X_T = -\|\vec{T}\| \cos \beta = 3260 \cos 21,3^\circ = 3037,2 \text{ daN}$$

$$Y_T = \|\vec{T}\| \sin \beta = 3260 \sin 21,3^\circ = 1186,5 \text{ daN}$$

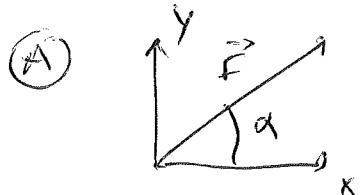
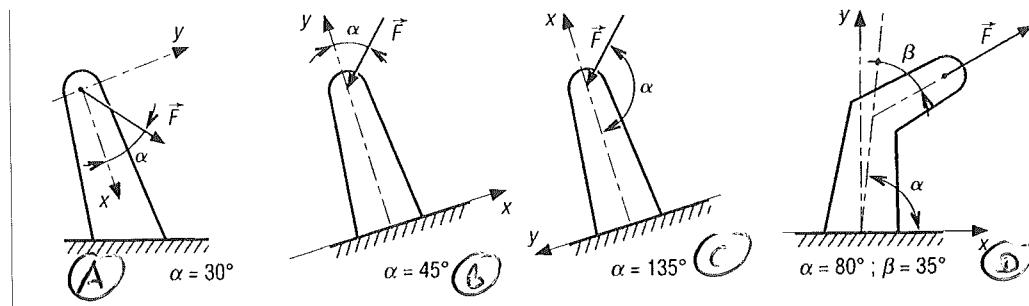
$$\vec{P} \begin{pmatrix} 0 \\ -2700 \end{pmatrix}$$

$$\vec{A} + \vec{T} + \vec{P} \quad \left| \begin{array}{l} 3041 - 3037,2 = 3,8 \text{ daN} \\ 1520,5 + 1186,5 - 2700 = -265 \text{ daN} \end{array} \right.$$

La somme vectorielle n'est pas nulle.

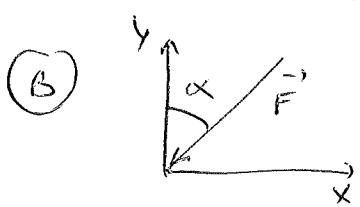
(7)

EXERCICE 4.

Écrire les coordonnées cartésiennes X_F et Y_F des forces \vec{F} en fonction du module et des angles α et $\|\vec{F}\| = 1000 \text{ N}$ dans les quatre cas.

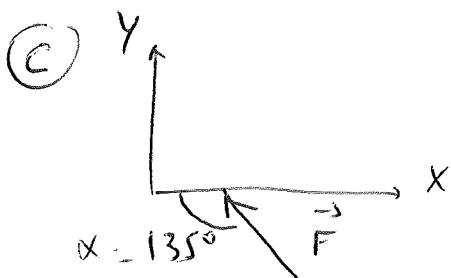
$$Y_F = F \cos \alpha = 1000 \cos 30^\circ = 866 \text{ N}$$

$$Y_F = F \sin \alpha = 1000 \sin 30^\circ = 500 \text{ N}$$



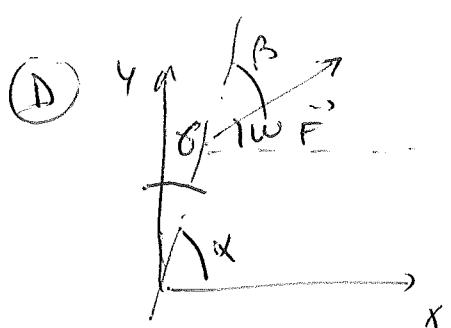
$$X_F = -F \cos(\pi/2 - \alpha) = -1000 \cos 45^\circ = -707 \text{ N}$$

$$Y_F = -F \sin(\pi/2 - \alpha) = -1000 \sin 45^\circ = -707 \text{ N}$$



$$X_F = -F \cos(\pi - \alpha) = -1000 \cos(45^\circ) = -707 \text{ N}$$

$$Y_F = F \sin(\pi - \alpha) = 1000 \cos(45^\circ) = 707 \text{ N}$$



$$\begin{aligned} \theta &= \frac{\pi}{2} - \alpha \\ w &= \frac{\pi}{2} - (\beta + \theta) \\ &= \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha + \beta\right) \\ &= \frac{\pi}{2} - \frac{\pi}{2} + \alpha - \beta \end{aligned}$$

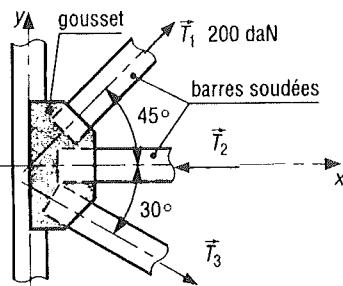
$$w = \cancel{\frac{\pi}{2}} \alpha - \beta = 80^\circ - 35^\circ = 45^\circ$$

$$X_F = F \cos(w) = 1000 \cos 45^\circ = 707 \text{ N}$$

$$Y_F = F \sin(w) = 1000 \sin 45^\circ = 707 \text{ N}$$

8

EXERCICE 5.

Déterminer les composantes X_{T1} et Y_{T1} de la tension (force) \vec{T}_1 de la barre 1.Déterminer X_{T3} et $\|\vec{T}_3\|$ si $Y_{T3} = 100 \text{ daN}$,Déterminer \vec{T}_2 si $X_{T1} + X_{T2} + X_{T3} = 0$ 

$$X_{T1} = T_1 \cos 45^\circ = 200 \cos 45^\circ = 141,4 \text{ daN}$$

$$X_{T2} = T_1 \sin 45^\circ = 200 \sin 45^\circ = 141,4 \text{ daN}$$



$$\frac{Y_{T3}}{\|\vec{T}_3\|} = \sin 30^\circ$$

$$\Rightarrow \|\vec{T}_3\| = \frac{Y_{T3}}{\sin 30} = \frac{100}{\sin 30} = 200 \text{ daN}$$

$$\frac{Y_{T3}}{X_{T3}} = \tan 30^\circ \Rightarrow X_{T3} = \frac{Y_{T3}}{\tan 30} = \frac{100}{\tan 30} = 173,2 \text{ daN}$$

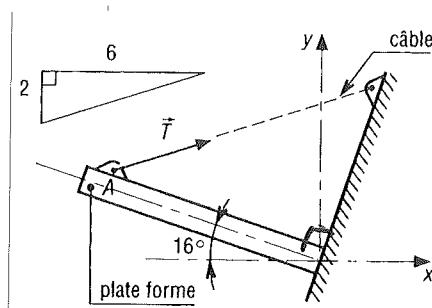
$$X_{T1} + X_{T2} + X_{T3} = 0$$

$$\Rightarrow X_{T2} = -X_{T1} - X_{T3} = -173,2 - 141,4 = -314,6 \text{ daN}$$

$$\vec{T}_2 \begin{pmatrix} -314,6 \text{ daN} \\ 0 \end{pmatrix} \quad \|\vec{T}_2\| = \sqrt{(-314,6)^2 + 0^2} = 314,6 \text{ daN}$$

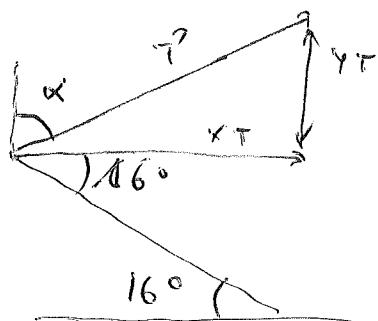
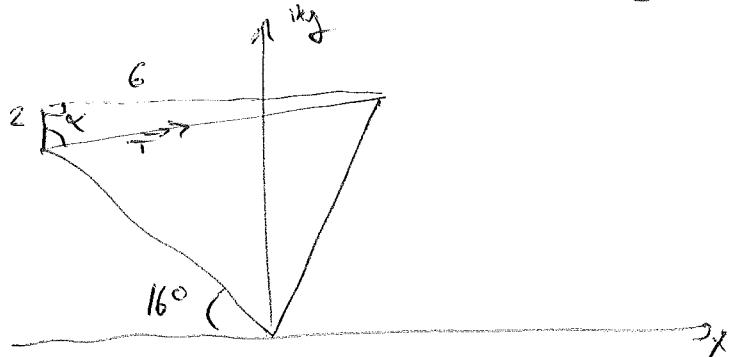
EXERCICE 6.

Sachant que la composante X_T de la tension \vec{T} du câble en A est de 90 daN, déterminer Y_T et T .



(g)

$$\alpha = \tan^{-1} \frac{6}{2} = 71,6^\circ$$



$$\frac{Y_T}{X_T} = \tan\left(\frac{\pi}{2} - \alpha\right)$$

$$\Rightarrow Y_T = X_T \tan\left(\frac{\pi}{2} - \alpha\right)$$

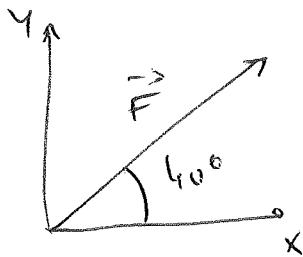
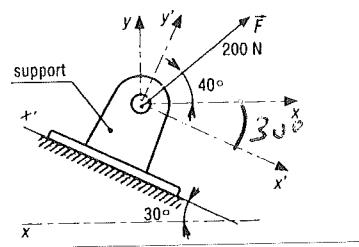
$$= g_0 \times \tan 18,4^\circ$$

$$= 30 \text{ daN} \quad \Rightarrow \vec{T} \Big| \begin{matrix} 30 \\ 30 \end{matrix}$$

$$\|\vec{T}\| = \sqrt{g_0^2 + 30^2} = 95 \text{ daN}$$

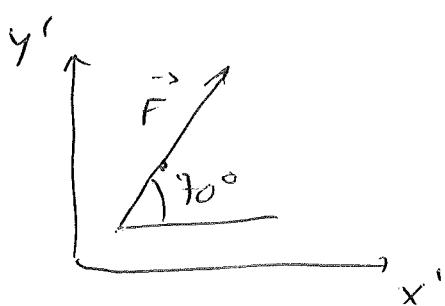
EXERCICE 7.

Déterminer les coordonnées de \vec{F} dans le système (x, y) puis dans le système (x', y').



$$X_F = F \cos 40^\circ = 200 \cos 40^\circ = 153,2 \text{ N}$$

$$Y_F = F \sin 40^\circ = 200 \sin 40^\circ = 128,6 \text{ N}$$

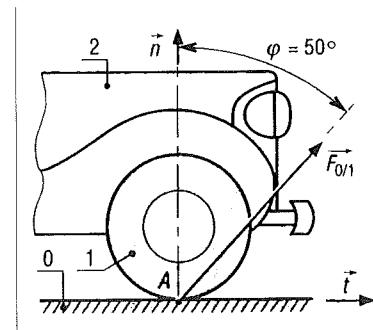


$$X'_F = F \cos 70^\circ = 200 \cos 70^\circ = 68,4 \text{ N}$$

$$Y'_F = F \sin 70^\circ = 200 \sin 70^\circ = 188 \text{ N}$$

EXERCICE 8.

L'action de la route O sur la roue motrice 1 est schématisée par la force $\vec{F}_{0/1}$. Si l'effort normal $\vec{N}_{0/1}$ suivant \vec{n} a pour valeur 400 daN, déterminer $\vec{F}_{0/1}$ et $\vec{T}_{0/1}$ (suivant \vec{t}) sachant que $\vec{F}_{0/1} = \vec{N}_{0/1} + \vec{T}_{0/1}$



$$N_{0/1} / F_{0/1} = \cos 50^\circ$$

$$\Rightarrow F_{0/1} = \frac{N_{0/1}}{\cos 50^\circ} = \frac{400}{\cos 50^\circ} = 622,3 \text{ daN}$$

$$\tan 50^\circ = \frac{T_{0/1}}{N_{0/1}} \Rightarrow T_{0/1} = \tan 50^\circ \cdot N_{0/1} = \tan 50^\circ \times 400 = 476,7 \text{ daN}$$